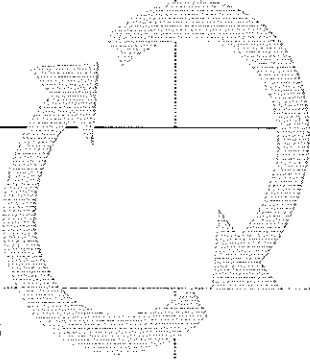


LESSON PLANNING FRAMEWORK

PLANNING FOR THE HIGH-COGNITIVE ENACTMENT OF A HIGH-COGNITIVE MATHEMATICS LESSON

Regular use of this framework – alone and with colleagues – fosters productive professional habits of mind and practice

| | | | |
|--|---|--|----------------|
| DO THE MATH | | | |
| DRAFT THE LESSON SUMMARY & FLOW | | | |
| ANTICIPATE STUDENT STRATEGIES & CONCEPTIONS | | | |
| PLENARY | SET UP THE TASK | STUDENTS | TEACHER |
| INDIVIDUAL & SMALL GROUP | MONITOR & SUPPORT STUDENT THINKING | STUDENTS | TEACHER |
| PLENARY | SELECT & SEQUENCE |  | |
| | CONNECT & EXTEND | | |
| COLLECT ARTIFACTS & LOOK AHEAD | | | |



LESSON SUMMARY

Lesson Title/Description and Source:

Mathematical Trajectory. *How is this lesson situated in the bigger mathematical storyline that is developing across the unit? year? and beyond?*

Core Mathematical Idea(s) That All Learners Will Understand Today. *What are your specific mathematical goals for today's lesson? What do you intend all students will know and understand about mathematics as a result of today's lesson?*

Evidence. *How will you know that all learners understand? What evidence will you collect?*

Exit Task. *What mathematical ideas will be the focus of your exit task – to inform your instructional next steps?*

Secondary Mathematical Focus(es). *What other mathematical ideas will be developed/used within the lesson?*

Prior Knowledge. *In what ways does the lesson build on students' previous knowledge? What definitions, concepts, or ideas do students need to know in order to begin to work on the task?*

Metacognition. *On what mathematical ideas will you focus students' reflections about their thinking and self-assessments of their understanding, progress, and needs? Where in the lesson will these be most productive?*

Teacher Reflection Tool Instructional Focus. *Which one or two questions from the Teacher Reflection Tools will be the focus of your work on your teaching practice during this lesson?*

Relevant Standards. *What standards (e.g., school, district, state, national) will be addressed during this lesson?*



LESSON FLOW

Lesson Title/Description and Source:

LAUNCH/SET-UP See the "Set Up the Task" section of the Expanded Lesson Planning Framework (LPF)

Grouping: *Plenary*

Time: _____

Materials: _____

Actions*:

EXPLORE/INVESTIGATE See the "Monitor and Support Student Thinking" section of the Expanded LPF

Grouping (circle all that apply): *Individual, Pairs, Groups of __, Plenary*

Time: _____

Are there times you will move in and out of plenary during this part of the lesson? If so, when? _____

Materials: _____

Actions*:

SUMMARIZE/CLOSE See the "Select, Sequence, Connect, and Extend" section of the Expanded LPF

Grouping: *Plenary*

Time: _____

Materials: _____

Actions*:

Exit Task:

* Flag adaptations you make to the published task to assure high cognitive student interaction. As a useful guide for making such adaptations, the *Teacher Reflection Tool—Worthwhile Mathematical Tasks* provides research-based high-cognitive outcomes for learners.

EXPANDED LESSON PLANNING FRAMEWORK - PAGE 1

PLANNING FOR THE HIGH-COGNITIVE ENACTMENT OF A HIGH-COGNITIVE MATHEMATICS LESSON

Regular use of this framework – alone and with colleagues – fosters productive professional habits of mind and practice

| | | | |
|--|--|----------------------------|---|
| DO THE MATH | <p>The point here is to examine your mathematical thinking about the task.</p> <ul style="list-style-type: none"> • What is the mathematical task on which students will be working? (Note: in this context, a “task” refers to a problem and/or set of related problems that center on the big ideas on which this lesson focuses. It is assumed the lesson/task, as designed, is high cognitive.) • What are all the ways the task can be approached/solved correctly? Why does each strategy work? • Which strategies/ideas can/cannot be generalized and why or why not? | | |
| DRAFT THE LESSON FLOW & SUMMARY | <ul style="list-style-type: none"> • LESSON FLOW: Briefly describe the planned flow of the lesson (e.g., give a bulleted list/outline of the lesson activities). Note any changes from the published version that were made to increase/maintain the level of cognitive demand. • LESSON SUMMARY: Complete the Lesson Summary, which clarifies the mathematical and pedagogical goals and focuses of the lesson. • For lessons involving a math coach or observers, be prepared to respond to inquiries about the Lesson Summary, Lesson Flow, and questions from this Expanded Lesson Planning Framework. | | |
| ANTICIPATE STUDENT STRATEGIES & CONCEPTIONS | <ul style="list-style-type: none"> • What correct and incorrect student strategies, conceptions, and responses do you anticipate? What’s your reasoning? • What mathematical concepts, processes, and ideas will students understand and struggle with? What’s your reasoning? | | |
| PLENARY SESSION | <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td data-bbox="256 1052 443 1932" style="text-align: center; vertical-align: middle;"> SET UP THE TASK </td> <td data-bbox="443 1052 1453 1932"> <p>GENERAL QUESTIONS ABOUT THE SET-UP</p> <ul style="list-style-type: none"> • What resources or tools will students have available to use in their work on the task? • How will the students work during the lesson – independently, in small groups, in pairs, and/or in plenary? When will groupings change? Why? • What other group structures, strategies, and status treatments are planned for the lesson? What are your rationale? • Will there be public records of students’ thinking? If so, how will these be displayed and used to support learning? <hr/> <p>STUDENTS</p> <ul style="list-style-type: none"> • What will students say/do that will indicate they understand the requirements of the task? • How, and for how long, will students engage privately with the task before making their thinking public in pairs, small groups, and/or the large group? • What will students say/do that suggests they may misunderstand the intent/requirements of the task? <p>TEACHER</p> <ul style="list-style-type: none"> • How do you intend to introduce the task to the students? Specifically, how will you word the task (keeping the cognitive demand high)? • How will you present it to the students (e.g., orally, in writing, at the overhead, on a hand-out, etc.)? • What will you do to foster thoughtful and accountable private think time for reflection about the mathematical task? • What will you do/ask to help students access their prior knowledge? • What will you do if a student does not know how to begin to solve the task or has misinterpreted the task? How will you intervene without reducing the cognitive demand of the task? </td> </tr> </table> | SET UP THE TASK | <p>GENERAL QUESTIONS ABOUT THE SET-UP</p> <ul style="list-style-type: none"> • What resources or tools will students have available to use in their work on the task? • How will the students work during the lesson – independently, in small groups, in pairs, and/or in plenary? When will groupings change? Why? • What other group structures, strategies, and status treatments are planned for the lesson? What are your rationale? • Will there be public records of students’ thinking? If so, how will these be displayed and used to support learning? <hr/> <p>STUDENTS</p> <ul style="list-style-type: none"> • What will students say/do that will indicate they understand the requirements of the task? • How, and for how long, will students engage privately with the task before making their thinking public in pairs, small groups, and/or the large group? • What will students say/do that suggests they may misunderstand the intent/requirements of the task? <p>TEACHER</p> <ul style="list-style-type: none"> • How do you intend to introduce the task to the students? Specifically, how will you word the task (keeping the cognitive demand high)? • How will you present it to the students (e.g., orally, in writing, at the overhead, on a hand-out, etc.)? • What will you do to foster thoughtful and accountable private think time for reflection about the mathematical task? • What will you do/ask to help students access their prior knowledge? • What will you do if a student does not know how to begin to solve the task or has misinterpreted the task? How will you intervene without reducing the cognitive demand of the task? |
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EXPANDED LESSON PLANNING FRAMEWORK - PAGE 2

INDEPENDENT AND/OR SMALL GROUP WORK

**MONITOR
& SUPPORT
STUDENT
THINKING**

STUDENTS While students are working individually and/or in small groups:

- How will students interact with each other about their mathematical thinking? What will you see and hear that tells you they are listening/inquiring to understand and build on each other's thinking?

- Specifically, what will students say/do that lets you know they are/are not making sense of the task? What correct and incorrect mathematical reasoning and justifications will students provide for their ideas and methods?

- What do you expect will be the boundaries of students' understandings?

- What will students say/do as indicators the task has not engaged their thinking? What might be the reasons for such disengagement?

- What will students say/do that will suggest they view mathematical reasoning and justification as the source of mathematical authority (vs status)?

- What will students say or do that shows they do/do not trust in their own capacity to think and reason mathematically (about the ideas at hand or in general)? How will they demonstrate they are accountable to the group and the content?

- What student responses might prompt you to call for a plenary?

- Specifically, what will students say/do that will indicate they are extending their thinking to new contexts, concepts, and/or methods and to the general case?

TEACHER While you monitor students working individually and/or in small groups:

- What specific questions, statements, and actions will you use to encourage students to share their thinking with each other, to build on each other's ideas, and/or to assess their understanding of each other's ideas?

- What questions, statements, and actions will you use to focus students' thinking and foster sensemaking – without diminishing the cognitive demands of the task?

- What questions, statements, and actions will you use to elicit and assess students' understanding of key mathematical ideas, processes, problem-solving strategies, and/or mathematical representations (without diminishing the cognitive demands of the task)? That is, how will you press for the depth and boundaries of students' thinking – how will you uncover their understandings as well as the nature and roots of their misconceptions and errors in reasoning?

- What will you do if a student finishes the task almost immediately, becomes bored or disruptive, and/or focuses on non-mathematical aspects of the activity (e.g., decorating a poster)?

- How will you respond to correct/incorrect strategies, conceptions, justifications, and answers – while maintaining cognitive demands and reinforcing the notion that authority for the correctness and sensibility of an idea resides in mathematical argumentation and reasoning?

- How will you assure that each student is individually accountable for contributing to and understanding her/his group's mathematical discourse, strategies, solutions, and public work?

- How will you keep track of student strategies, understandings, and struggles (for formative assessment purposes and so you can select and sequence student strategies for a plenary)?

- What specific questions, statements, and actions will you use to advance students' understanding of the mathematical ideas and to promote generalization?

EXPANDED LESSON PLANNING FRAMEWORK - PAGE 3

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| PLENARY SESSION(S) | <p>ABOUT PLENARIES: <i>Besides a plenary to set up the lesson and at the end of the class period for summarizing and connecting the day's learning, are you planning for other plenaries during the lesson (e.g., to address confusions or to provide scaffolds during the main exploration/investigation)? What student thinking might lead you to call for an impromptu plenary?</i></p> | |
| | <p>SELECT</p> <ul style="list-style-type: none"> • During each plenary, which anticipated student strategies and other mathematical ideas will be shared and how will they be shared? • What will you do if key ideas you want to come up don't come up? How will you raise these ideas for consideration at a high cognitive level? | |
| | <p>SEQUENCE</p> <ul style="list-style-type: none"> • How will you sequence the order in which student (and other) strategies/solutions are presented during each plenary? • How will this sequencing support/advance students' understanding of the mathematical ideas on which the lesson focuses? | |
| | <table border="0" style="width: 100%;"> <tr> <td style="width: 50%; vertical-align: top;"> <p>STUDENTS For each plenary that you call, consider these questions:</p> <ul style="list-style-type: none"> • What will students say/do to show they are listening to understand, expanding on, debating, and questioning the ideas, strategies, and solutions shared by their classmates? • How will students show they respect each other's right and capacity to solve problems? • What mathematical patterns, connections, similarities, and differences will students identify among their solutions? • What will be students' conjectures, justifications, and generalizations related to the ideas at hand? • During the closing plenary, what student-based evidence will suggest that the lesson has/has not leveraged understanding of the core mathematical ideas that are the goals for the lesson? </td> <td style="width: 50%; vertical-align: top;"> <p>TEACHER For each plenary that you call, consider these questions:</p> <ul style="list-style-type: none"> • What structures/strategies will you use to encourage students to listen to understand, expand on, debate, and question the ideas and solutions shared by their classmates – while respecting each others' right and capacity to solve problems? • What questions will you ask to encourage students to analyze and compare the ideas and solutions that are presented? • How will you elicit mathematical conjectures, justifications, and generalizations about the ideas at hand? • How/where will you document, create, and/or display a public record of student strategies, conjectures and generalizations? How do you plan to use these ideas to advance students' thinking? • During the closing plenary, what will you do to determine whether the lesson has leveraged understanding of the core mathematical ideas you have identified as goals for the lesson? </td> </tr> </table> <p>CONNECT & EXTEND</p> | <p>STUDENTS For each plenary that you call, consider these questions:</p> <ul style="list-style-type: none"> • What will students say/do to show they are listening to understand, expanding on, debating, and questioning the ideas, strategies, and solutions shared by their classmates? • How will students show they respect each other's right and capacity to solve problems? • What mathematical patterns, connections, similarities, and differences will students identify among their solutions? • What will be students' conjectures, justifications, and generalizations related to the ideas at hand? • During the closing plenary, what student-based evidence will suggest that the lesson has/has not leveraged understanding of the core mathematical ideas that are the goals for the lesson? |
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EXPANDED LESSON PLANNING FRAMEWORK - PAGE 4

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| COLLECT ARTIFACTS & LOOK AHEAD | <p>STUDENTS</p> <ul style="list-style-type: none"> • <i>What are likely/intended responses to the reflection prompts and student self-assessment(s) you pose during and/or at the end of the lesson?</i> • <i>What student thinking do you anticipate will be revealed by the other artifacts you collect during and/or at the end of the lesson?</i> • <i>What will students say/do that tells you they understand what is expected for the homework?</i> | <p>TEACHER</p> <ul style="list-style-type: none"> • <i>During/after the lesson, what will you do to engage students in reflection and self-assessment about the development of their thinking and/or their growth as engaged mathematical thinkers? Specifically how will you word, present, and collect these student reflections and self-assessments?</i> • <i>How will you use the student reflections and self-assessments to inform your instruction?</i> • <i>What other artifacts of student thinking will you collect at the end of the lesson to inform your planning? What work from the lesson will you collect?</i> • <i>If you assign an entry or exit task or a closing reflection about the mathematical ideas in the lesson, specifically how will it be worded? presented? collected?</i> • <i>What homework will you assign to focus, assess, and/or advance students' thinking about the key mathematical ideas embedded in this lesson?</i> • <i>What will you do during your next lesson(s) that will build on this lesson?</i> |
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